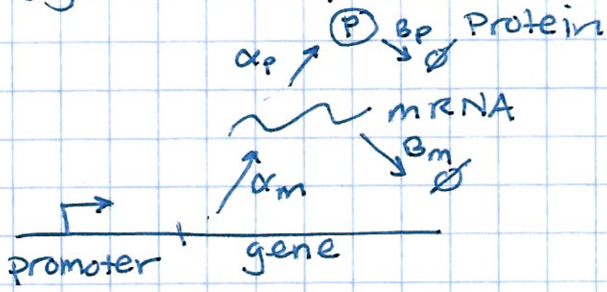
MODELING GENE EXPRESSION WITH DIFFERENTIAL EQNS

CONSTITUTIVE GENE EXPRESSION

(eg. CONSTANT, NO REGULATION)



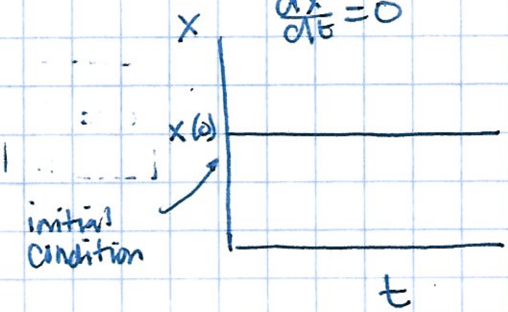
let m = mRNA, p = protein

ORDINARY DIFFERENTIAL EQUATIONS (ODEs)

$\frac{dx}{dt}$ = rate of change of x over time

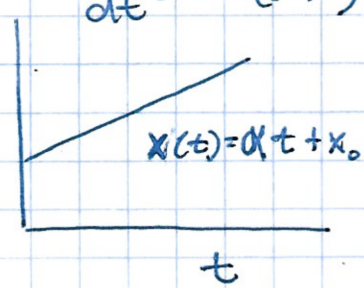
STATIC

$\frac{dx}{dt} = 0$



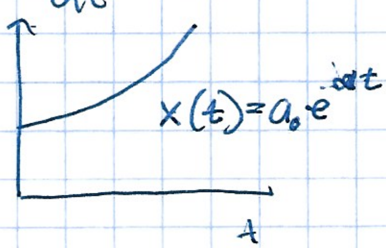
ZEROTH ORDER

$\frac{dx}{dt} = \alpha \quad (\alpha > 0)$



FIRST ORDER

$\frac{dx}{dt} = \alpha x \quad (\alpha > 0)$



X CAN BE A BIOCHEMICAL SPECIES (mRNA, protein).

LAW OF MASS ACTION

$$X + Y \underset{k_2}{\overset{k_1}{\rightleftharpoons}} Z$$

THE RATE OF A REACTION IS PROPORTIONAL TO THE PRODUCT OF THE CONCENTRATION OF THE REACTANTS

$$\frac{dx}{dt} = -k_1 X \cdot Y + k_2 Z$$

$$\frac{dY}{dt} = -k_1 X Y + k_2 Z$$

$$\frac{dZ}{dt} = k_1 X Y - k_2 Z$$

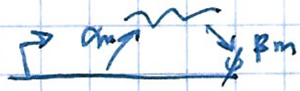
FOR DIMERS,

$$X + X \xrightarrow{k} Y$$

$$\frac{dX}{dt} = -kx^2 \qquad \frac{dY}{dt} = kx^2$$

IN GENERAL, THE EXPONENT CORRESPONDS TO THE # OF MOLECULES ($n=2$).

OK SO LET'S APPLY THIS TO mRNA & PROTEIN



__mRNA__

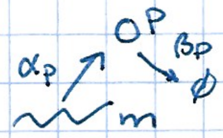
$$\frac{dm}{dt} = \alpha_m - \beta_m m$$

synthesis constant

degradation constant

· USE LAW OF MASS ACTION
· ASSUME DNA CONSTANT

__protein__



$$\frac{dp}{dt} = \alpha_p m - \beta_p p$$

__FINAL MODEL__

$$\frac{dm}{dt} = \alpha_m - \beta_m m$$

$$\frac{dp}{dt} = \alpha_p m - \beta_p p$$

NOW WE NEED: · PARAMETER VALUES
· INITIAL CONDITIONS

BUT EVEN BEFORE WE SOLVE $m(t)$, $p(t)$
WE CAN FIND STEADY-STATE RELATIONSHIP

$$\frac{dm}{dt} = 0 = \alpha_m - \beta_m m_{ss} \qquad m_{ss} = \alpha_m / \beta_m$$

$$\frac{dp}{dt} = 0 = \alpha_p m_{ss} - \beta_p p_{ss} \qquad p_{ss} = \frac{\alpha_p m_{ss}}{\beta_p} = \frac{\alpha_p \alpha_m}{\beta_p \beta_m}$$

EVEN WITHOUT SOLVING WE CAN SEE HOW PARAMETERS WILL IMPACT THE STEADY STATE BEHAVIOR OF THE SYSTEM.